Houghton Mifflin

Math Expressions

Math Talk and Helping Community in *Math Expressions*

Dr. Karen Fuson

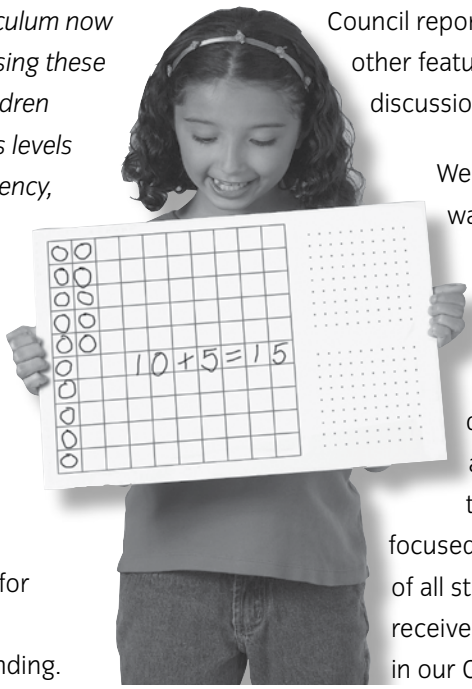
Program Author of ***Math Expressions*** and Professor Emeritus, Learning Sciences, School of Education and Social Policy, Northwestern University

This is the second article describing the classroom structures found in the **Math Expressions** *program. These structures are Building Concepts, Math Talk, Student Leaders, Quick Practice, and Helping Community. Though we discuss the five structures in separate author papers, they interact synergistically in the classroom. The* Children's Math Worlds Research Project (CMW) *that developed the curriculum now called* **Math Expressions** *found that using these structures in the classroom enables children from all backgrounds to learn ambitious levels of mathematics with understanding, fluency, and confidence.*

WHAT IS MATH TALK?

The National Council for Teachers of Mathematics (NCTM) Standards and two recent National Research Council reports *(Adding It Up: Helping Children Learn Mathematics* and *How Students Learn: Mathematics in the Classroom)* emphasize the need for students to discuss their mathematical thinking as a way to increase understanding.

The CMW Research Project that developed the ***Math Expressions*** program undertook research to identify crucial aspects of such discussion and to identify levels through which teachers could move from traditional teacher-focused teaching to productive student-to-student discussion monitored and supported by the teacher, as recommended by the National Resource Council reports. We also sought to identify other features that could support effective discussions.



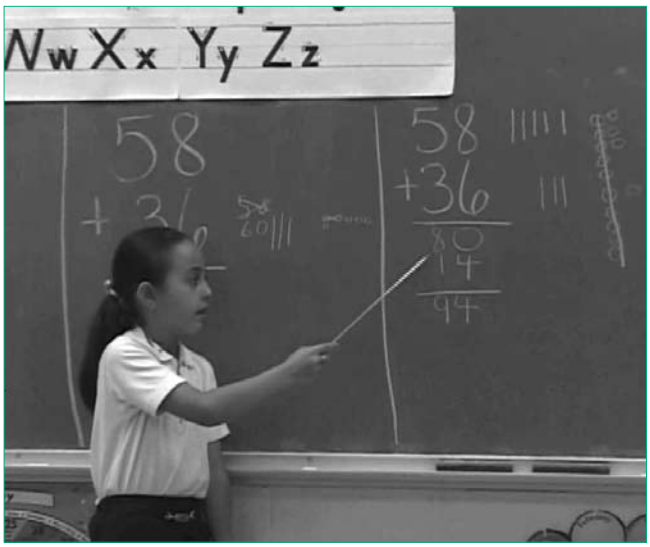
We found that the term *Math Talk* was effective in communicating with teachers and students the focus on discussion we desired. Our view was that Math Talk is an instructional conversation directed by the teacher but with as much direct student-to-student talk as possible. Math Talk is focused on developing the understanding of all students in the class. However, we received reports of teachers not involved in our CMW project who were using Math

Talk in other ways. Some teachers just had students take turns telling about their problem-solving methods or participate in undirected talk rather than actually analyzing and comparing methods. Some teachers did not seem to be trying to help students move along a learning path toward more effective methods of problem solving.

In true Math Talk, teachers create a Math Talk Inquiry environment and encourage constructive discussion of problem-solving methods through well-defined classroom activity structures, based on the four crucial aspects of Math Talk. Table 1 (see insert) describes the four aspects of Math Talk (questioning, explaining math thinking, source of math ideas, and responsibility for learning), describes the levels of Math Talk, and summarizes roles of the teacher and of the students in the highest level, Level 3. This table was developed initially from a case study of a teacher who moved from Level 1 to Level 3 with the support of a researcher. Then the table was modified as we studied other teachers moving through the levels and was published as a research paper (Hufferd-Ackles, Fuson, & Sherin, 2004). By now many teachers have used the table to help them move through the levels. There are also many Math Talk boxes in the ***Math Expressions*** Teacher's Editions that provide specific help in doing Math Talk.

LEARNING MATH TALK

Effective Math Talk cannot be implemented into a classroom overnight. A teacher must work his or her students up to Level 3 Math Talk over time. It often takes two or three months to build a classroom up to Level 3 if students are not familiar with Math Talk from the start. Initially the teacher and more advanced students will do a lot of modeling of solving and explaining for other students. In the beginning, teachers also concentrate on building listening skills by asking students to repeat a problem, question, or explanation in their own words. Teachers also elicit questions from students. These may apply to any topic (e.g., *How does your method relate to the method that Sam just explained?* or *Why did you do this method?*) or may apply specifically to a given math topic. In the latter case, the teacher needs to model some of these questions to use new math vocabulary, though often more advanced students can also think of such questions. Students need to learn to stand beside their drawing and numerical work and point to parts of it with a pointer as they explain. Students often initially explain only one part of their thinking or explain it incompletely. Using questions can help expand a student's explanation. The teacher or another student may also expand or clarify the student's explanation



through questions, though always checking with the original explainer to be sure that the assistance is what the student meant.

In addition to helping students learn Math Talk methods, teachers often need to adjust to the Math Talk structures themselves. They must learn to wait patiently and use a "bite your tongue" strategy to allow student talk to emerge. They must also physically move to the side or back of the room to facilitate student-to-student talk so that the explainer looks at classmates and not at the teacher. From the side or back of the room, a small gesture can be used to remind the explainer to look at classmates rather than at the teacher. Teachers can provide community assistance by asking explainers if they need help, but they also need to allow wait time before doing so. Shy students initially may need the presence of a friend at the board with them even if the friend does not help with the explaining. As teachers provide the space and support for students' voices to emerge, they often report being frequently amazed by the mathematical thinking their students are able to express.

ENGAGING IN MATH TALK

We found that two kinds of Solve, Explain, Question, and Justify classroom activity structures were very effective in engaging all students in Math Talk. In both structures, all students solve problems simultaneously. In the first structure, as many students as possible go to the board to solve a problem while the rest of the students work at their seats. Then the teacher selects two or three students from the board who have interesting solutions, or need the chance to explain their work, to talk about their solution. Only two or three students need to explain their work since usually students cannot maintain concentration for more than two or three discussions of the same problem. Next a different group of students goes to the board to solve the next problem. This process is very motivating. Most students enjoy solving problems at the board even if they do not get the chance to explain their work. While the students are working at the board, the teacher has a chance to see how solutions evolve. The teacher also gets a good feeling for how individual students are doing. In one class period many or even all of the students can get a turn at the board.

The second effective classroom structure allows every student in the classroom to explain his or her solution. Every student solves a problem at his or her seat. Then two or three students are selected by the teacher to go to the board to draw their solutions. The students left at their desks then pair up and explain their solutions to each other. Then the class discusses the solutions of those students at the board. Students at their seats can write their solutions on paper, which can be picked up and skimmed later by the teacher to see how students are doing. Another option is to have students at their desks solve problems on the large (12" by 19"), individual ***Math Expressions*** dry-erase boards called MathBoards. These MathBoards permit the teacher to send any additional student to the board for another explanation of the discussed problem because the drawings on the MathBoard are large enough to be seen by classmates.

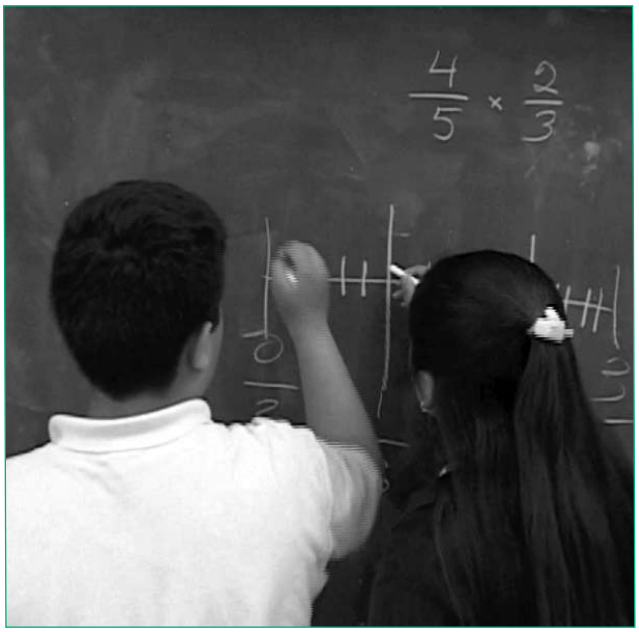
An important feature of both of these classroom structures is that no class learning time is lost. In other approaches, when students are sent to the board to draw their work, the rest of the class remains at their seats doing nothing. In the case of these ***Math Expressions*** structures, the students at their desks are just as involved in the problem solving as those at the board. Sometimes a step-by-step variation of these activity structures is helpful. Teachers can have each student explain one step of a solution at a time until a final solution is reached. Another method is to put students in pairs. The pairs can solve together and explain their work, with the less-advanced student

explaining first and the other, more-advanced student expanding and clarifying as needed.

The opportunity for all students to explain their math thinking over time is especially valuable for students learning English, as well as for native speakers advancing their verbal communication skills. Ultimately, developing understanding and verbal communication will aid all students in their future education and careers. In addition to verbal communication, the use of math drawings is central to Math Talk. Math drawings can show the quantities in a computation and relate them to a written numerical method or can show the situation in a word problem. The math drawings help everyone understand the student's math thinking. The special learning supports on the MathBoards enable students to learn meaningful drawings rapidly and then the open space on the MathBoards is used for math drawings.

A NURTURING MEANING-MAKING MATH TALK COMMUNITY

The Math Talk Inquiry environment is also a Nurturing Meaning-Making Helping Community in which everyone is a teacher and a learner. This creates a secure base for learning and for Math Talk. It enhances everyone's mathematical understanding, competence, and confidence. Teachers build the Helping Community daily by helping students learn how to help each other at the board, in pairs or groups, or when working individually. ***Math Expressions*** was developed through 10 years of intensive classroom research in many classrooms with students from many different cultural and linguistic backgrounds. The Nurturing Meaning-Making Math Talk Helping Community enables students from all backgrounds to bring their family and cultural experiences into the classroom and be validated, affirmed, and understood.



The Math Talk approach used by ***Math Expressions*** supports deeper understanding and more complex language learning than other reform approaches because the ***Math Expressions*** program provides research-based learning paths that move all students forward. Students develop the prerequisite understandings so that they can invent interesting methods. Research-based accessible strategies are taught so that everyone has an effective method. These relate to common methods so that all methods can be discussed and related. Many students learn, relate, and explain multiple methods. Math drawings enable students to solve and explain more effectively and enable listeners to understand and question more effectively. This process of discussing the whole developmental range of solution methods permits differentiated instruction to occur in whole-class activities but also enables students to move forward to a mathematically-desirable effective method. ***Math Expressions*** truly supports teachers as they develop a Nurturing Meaning-Making Math Talk Inquiry Classroom.

Levels of Math Talk Learning Community: Teacher and Student Action Trajectories			
Components of the Math Talk Learning Community			
A. Questioning	**B. Explaining math thinking**	**C. Source of math ideas**	**D. Responsibility for learning**
Overview of shift over Levels 0–3: The classroom community grows to support students acting in central or leading roles, and shifts from a focus on answers to a focus on mathematical thinking.			
There is a shift from the teacher as questioner to the students and teacher as questioners.	The students increasingly explain and articulate their math ideas.	There is a shift from the teacher as the source of all math ideas to students' ideas also influencing the direction of lessons.	The students increasingly take responsibility for learning and evaluation of others and of themselves. Math sense becomes the criterion for evaluation.
Level 0: This is a traditional teacher-directed classroom with brief answer responses from students.			
Level 1: The teacher is beginning to pursue student mathematical thinking. The teacher plays a central role in the Math Talk community.			
Level 2: The teacher models and helps students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. The teacher physically moves to the side or back of the room and directs from there.			
Level 3: The teacher is a co-teacher and co-learner. The teacher monitors all that occurs and is still fully engaged. The teacher is ready to assist, but now in a more peripheral and monitoring role (coach and assister).			
The teacher expects students to ask one another questions about their work. The teacher's questions still may guide the discourse. Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions of each other and listen to responses. Many questions are "Why?" questions that require justification from the person answering. Students repeat their own or other students' questions until they are satisfied with the answers.	*The teacher follows along closely to student descriptions of their thinking, encouraging students to make their explanations more complete; he or she may ask probing questions to make explanations more complete. The teacher stimulates students to think more deeply about strategies.* The students describe more complete strategies; they ***defend and justify*** their answers with little prompting from the teacher. The students realize that other students will ask them questions, so they are motivated and careful to be thorough. Other students provide support with active listening.	*The teacher allows for contributions from students during his or her explanations; he or she lets the students explain and "own" new strategies. The teacher is still engaged and deciding what is important to continue exploring. The teacher uses student ideas and methods as the basis for lessons or mini-extensions.* The students contribute their ideas as the teacher or other students are teaching, confident that their ideas are valued. The students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.	*The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. He or she supports students as they help one another sort out misconceptions. He or she helps and/or follows up when needed.* The students listen to understand, then initiate clarifying other students' work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. The students assist each other in understanding and correcting errors.

Table 1

REFERENCES

Fuson, K. C., De La Cruz, Y., Smith, S., Lo Cicero, A., Hudson, K., Ron, P., & Steeby, R. (2000). Blending the best of the 20th century to achieve a Mathematics Equity Pedagogy in the 21st century. In M. J. Burke & F. R. Curcio (Eds.), *Learning Mathematics for a New Century,* (pp. 197–212). Reston, VA: NCTM.

Fuson, K. C. (2003). Developing mathematical power in whole number operations. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 68–94). Reston, VA: NCTM.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan & J. D. Bransford (Eds.), *How Students Learn: Mathematics in the Classroom* (pp. 217–256). Washington, DC: National Academy Press.

Fuson, K. C., & Murata, A. (in press). Integrating NRC principles and the NCTM Process Standards to form a Class Learning Path Model that individualizes within whole-class activities. *NCSM Journal of Mathematics Education Leadership.*

Hufferd-Ackles, K., Fuson, K., & Sherin, M. G. (2004). Describing levels and components of a math-talk community. *Journal for Research in Mathematics Education,* 35 (2), 81–116.

Kilpatrick, J., Sowder, J., & Findell, B. (Eds.) (2001). *Adding It Up: Helping Children Learn Mathematics.* Washington, DC: National Academy Press.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.