SS5 • Evaluating statements about enlargement

Mathematical goals

To enable learners to:

- explore the relationship between linear and area enlargement;
- substitute into algebraic statements;
- discuss some common misconceptions about enlargement.

Starting points

Most learners will have covered some aspects of enlarging shapes before, but they may not have explored the relationships between linear, area and volume enlargement. In this session learners sort statements into categories: always, sometimes or never true. Throughout their work on this activity, learners justify and explain their decisions using examples and counter-examples. They present their ideas to the rest of the group by means of a poster.

Materials required

- OHT of Sheet 1 – *Perimeters and areas*.

For each small group of learners you will need:

- Sheet 1 – *Perimeters and areas*;
- Card set A – *Doubling statements*;
- Card set B – *Diagrams*;
- large sheet of paper for making a poster;
- felt tip pen;
- glue stick;

and possibly:

- Sheet 2 – *Rep-tiles: enlarging different shapes*.

Time needed

At least 1 hour.

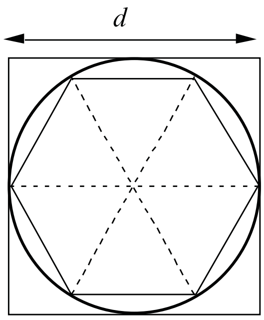
Suggested approach

Beginning the session

Ask learners, working alone, to complete Sheet 1 – *Perimeters and areas*. This helps them to review the meaning of area and perimeter and reminds them (with some justification) of the formulae πd and πr^2.

Whole group discussion

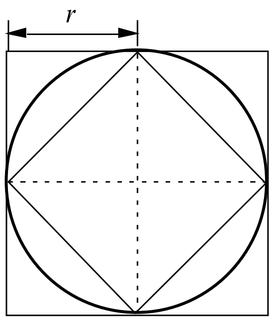
After a short while, hold a whole group discussion to review this work, using the OHT of Sheet 1 – *Perimeters and areas*.



Show the first drawing.

What is the perimeter of the square? ($4d$)
What is the perimeter of the hexagon? ($3d$)
The perimeter of the circle (or circumference) lies between $3d$ and $4d$. Does anyone know what it is?

The value is about $3.142 \times d$. The number 3.142 is only approximate. It is impossible to write the number down exactly, as it has an infinite number of decimal places with no pattern to them. We often use the letter π to represent this number.



Now show the second drawing.

What is the area of the large square? ($2r \times 2r = 4r^2$)
What is the area of the small square? (Half as much $= 2r^2$)
The area of the circle lies between $2r^2$ and $4r^2$

The area of the circle is about $3.142 \times r^2$ or πr^2, where π turns out to be the same number as before.

Working in groups

Organise learners into groups of two or three.

Give each group Card set A – *Doubling statements*, a large sheet of paper, a glue stick and a felt tip pen. Ask learners to divide their sheet into three columns and head the columns with the words 'Always true', 'Sometimes true', 'Never true'.

Talk through the following steps with learners, maybe using one or two examples from the sheet.

1. Choose a pair of statements

Choose a pair of statements about a particular shape. If you are confident, choose a harder shape; if you are less confident, choose the rectangle.

2. Look at a special case

Draw the shape you have chosen and give it some measurements. Draw a larger version with double the dimensions. Calculate areas and perimeters. Do the statements appear to be true?

3. If you think a statement is true

Try to show why the statement works by drawing or by using algebra. Let the lengths on the drawing be x and y for example, and calculate the other lengths in terms of these.

4. If you think a statement is false

Try to say how the statement should be changed so that it becomes true. Explain why you are sure that your version is true.

Learners should take it in turns to place a card in one of the columns and justify this response to their partner. Their partner must challenge them if the explanation has not been clear and complete. When the pair agrees, they should stick the card down and write examples and explanations to show the reasoning behind the decision.

It is not necessary for all learners to tackle every statement. Encourage groups to begin with statements at an appropriate level of difficulty.

Explain that learners will be expected to present their conclusions to the rest of the group, using their posters.

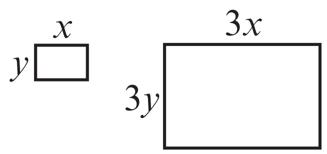
If learners get stuck, you may like to offer them an appropriate card from Card set B – *Diagrams*. These provide some helpful hints and constructions that will get them started.

When learners make rapid progress, encourage them to explore additional shapes and also explore what happens when the dimensions are trebled.

Reviewing and extending learning

Ask each pair of learners to present what they have found to the rest of the group, using their posters. As they do this, ask other learners for further examples and reasons. Encourage numerical, geometrical and algebraic reasoning.

For example, if someone has considered trebling the sides of a rectangle, one could compare the perimeters $2x + 2y$ and $6x + 6y$, and see that, whatever values x and y take, one perimeter will be three times as long as the other. Similarly, it is immediately clear that the area has increased by a factor of 9.



What learners might do next

Learners may like to look at the enlargement of other shapes. Sheet 2 – *Rep-tiles: enlarging different shapes* has been provided for this purpose.

Further ideas

This activity is about examining a mathematical statement and deciding on its truth or falsehood. This idea may be used in many other topics and levels. Examples in this pack include:

A4 Evaluating algebraic expressions;

SS4 Evaluating statements about length and area;

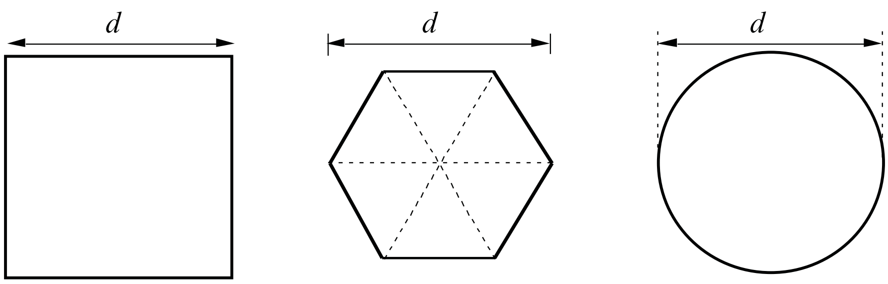
S2 Evaluating probability statements.

SS5 Sheet 1 – *Perimeters and areas*

How would you explain the terms 'perimeter' and 'area' to someone who has never heard of them?

Think about this question for a few minutes and note down your thoughts. Then tackle the questions below.

Perimeter



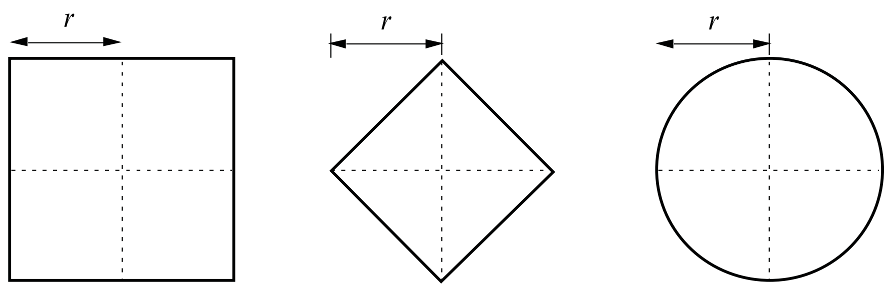
Which of these shapes has the greatest perimeter?

Which has the smallest perimeter?

Can you write each perimeter in terms of the letter d ?

If you can't do this exactly, make an estimate.

Area



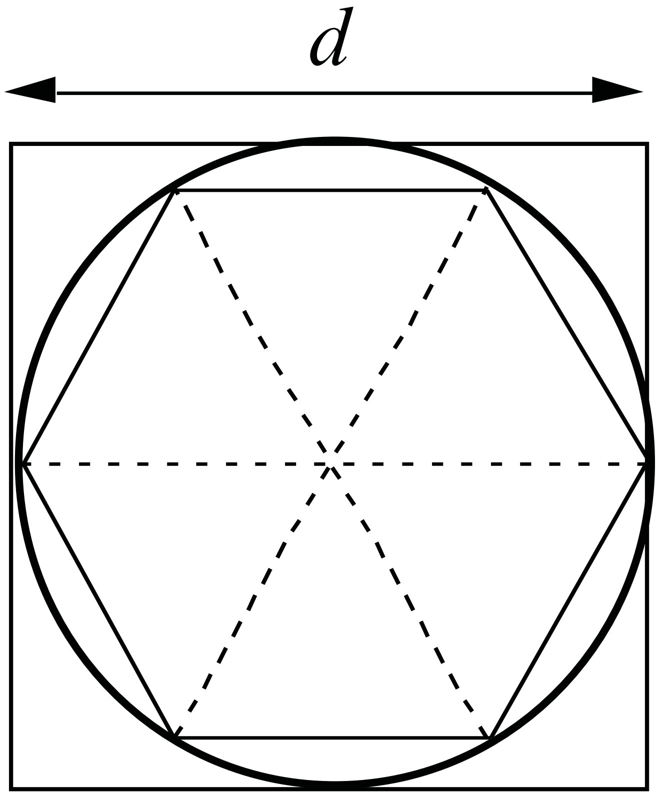
Which of these shapes has the greatest area?

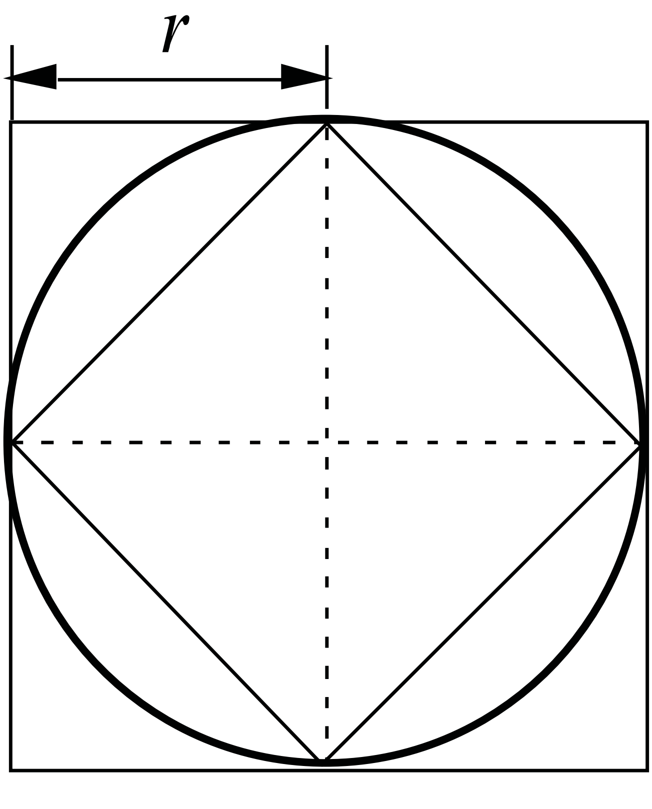
Which has the smallest area?

Can you write each area in terms of the length r ?

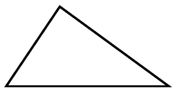
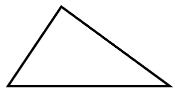
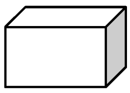
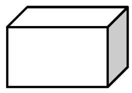
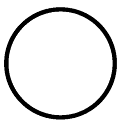
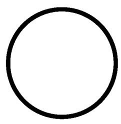
If you can't do this exactly, make an estimate.

SS5 OHT – *Perimeters and areas*



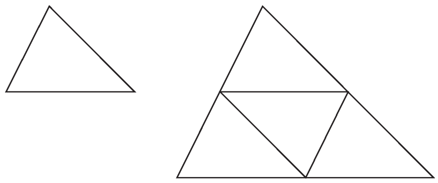


SS5 Card set A – *Doubling statements*

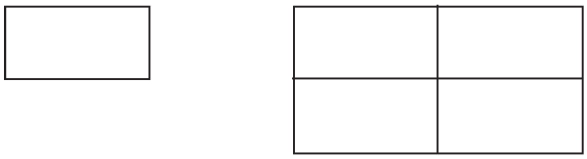
 **If you double the lengths of the sides of a rectangle, its perimeter doubles.**	 **If you double the lengths of the sides of a rectangle, its area doubles.**
 **If you double the lengths of the sides of a triangle, its perimeter doubles.**	 **If you double the lengths of the sides of a triangle, its area doubles.**
 **If you double the lengths of the edges of a cuboid, its volume doubles.**	 **If you double the lengths of the edges of a cuboid, its surface area doubles.**
 **If you double the radius of a circle, its area doubles.**	 **If you double the radius of a circle, its circumference doubles.**

SS5 Card set B – *Diagrams*

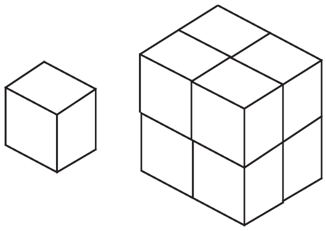
Triangles



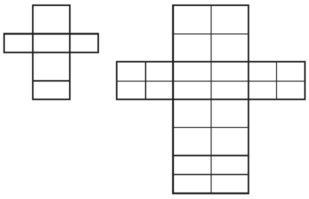
Rectangles



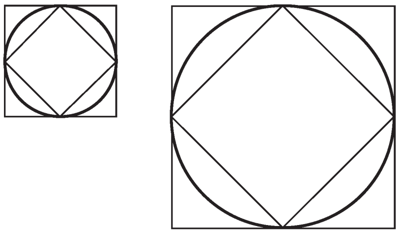
Cuboid volumes



Cuboid surface areas



Circles



SS5 Sheet 2 – *Rep-tiles: enlarging different shapes*

Each small tile can be tessellated to give a larger version of itself.

Consider how the lengths, perimeters and areas of the tiles change as they are enlarged and write about the relationships you find.